1. Rewrite each of the following sentences as suggested/indicated:

(i) 若天氣好，我去行山 (If weather is good then I go hiking)

(a) Rephrase the above sentence by using "whenever"

(ii) 凡中大学生必，是好人

(a) $\ldots \subseteq \ldots$, where the sets are defined by

$\ldots$

(b) If $\ldots$ then (用(a)定义之集合)

(c) $\ldots \in \ldots$ for $\ldots \in \ldots$

(d) $\ldots$, whenever $\ldots$

2 State the negation for each of (i), and (ii) of Q1.

3. Use axiom I show that, $\forall\, a, b \in \mathbb{R}$,

$$(a+b)^2 = a^2 + 2ab + b^2$$

and

$$(-a)^2 = a^2$$

other properties in I & II

4. Show by MI (or extended MI) that

$$1 < 2 < 3 < \cdots$$

and that

$$(n, n+1) \cap \mathbb{N} = \emptyset \quad \forall\, n \in \mathbb{N}.$$

Can you extend the results to $\mathbb{Z}$, integers?

Show further that if $j, k \in \mathbb{Z}$ are s.t. (such that)

$$j < k$$

then $j + 1 \le k$.

5. Show that $\max X$ (and similarly $\min X$) is unique when exists;

x' is called a largest element (maximal element) of X if

$x' \in X$ and

$x \leq x' \ \forall x \in X$.

Show that x'' is also a largest element of X then $x' = x''$.

6. What is meant that

(i) $\bar{x} \in \mathbb{R}$ is not a largest ele. of X

(ii) $\underline{x}$ is not a smallest ele. of X

7. A set Y of real numbers is said to be bounded above if there exists a real number u such that

$$\text{---} \leq u \quad \text{-----}.$$

(such an u is called an upper bound of Y).

Fill the blanks and state the negation (what is meant that Y is not bounded above)

$\hat{7}$. Do Q7 for bounded below.

8*. Provide a bounded (= bounded below and bounded above) set X of real ~~numbers~~ such that $\min X$, $\max X$ do not exist. Check your assertion.

9*(i) Show that $(\forall x \in \mathbb{R})$

$$x, -x \leq |x|$$

and that $x = |x|$ or $-x = |x|$.

(ii) Let $x, y \in \mathbb{R}$ and $0 < \alpha \in \mathbb{R}$.
Show that

$$|x| < \alpha \iff -\alpha < x < \alpha$$

$$|x-y| < \alpha \iff x-\alpha < y < x+\alpha$$

(The assumption $\alpha > 0$ is redundant)

10. Show, $\forall a, b \in \mathbb{R}$, that

$$|a \cdot b| = |a| \cdot |b|$$

and that

$$\big||a| - |b|\big| \leq |a \pm b| \leq |a| + |b|$$

(anti-triangle inequality & triangle inequality)

11*. We sometimes write

(the notation suggested by looking at the graphs of $x \mapsto \max\{f(x), g(x)\}$
$x \mapsto \min\{f(x), g(x)\}$
for real-valued functions f, g):

$$\left.\begin{array}{l} a \vee b := \max\{a, b\} \\ a \wedge b := \min\{a, b\} \end{array}\right\} \quad \forall a, b \in \mathbb{R}.$$

Show that, $\forall a, b \in \mathbb{R}$,

$$-(a \vee b) = (-a) \wedge (-b), \quad -(a \wedge b) = (-a) \vee (-b)$$

$$a \vee b = \frac{a+b-|a-b|}{2} \quad \left(\frac{和+差}{2} = 大数\right)$$

$$a \wedge b = \frac{(a+b)-|a-b|}{2} \quad \left(\frac{和-差}{2} = 小数\right)$$

小学時代的公式!

12. Let $\emptyset \neq B \subseteq \mathbb{R}$ and

$$-B := \{-b : b \in B\}.$$

Show that

(i) B is bounded below iff (if and only if) $-B$ is bounded above

(ii) $l \in \mathbb{R}$ is a l.b of B iff $-l$ is an u.b of $-B$